Name.. Number...

Gosford High School

HIGHER SCHOOL CERTIFICATE

2017 TRIAL EXAMINATION

Mathematics

- **General Instructions**
- Reading time – 5 minutes
- Working time – 3 hours
- Write using black pen
- Board-approved calculators may be used
- A reference sheet is provided at the back of this paper
- In Questions 11 – 16, show relevant mathematical reasoning and/or calculations

Total Marks – 100

Section I

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II

90 marks

- Attempt Questions 11 – 16
- Allow about 2 hours and 45 minutes for this section

Marks

Section I

10 Marks
Attempt Questions 1-10.
Allow about 15 minutes for this section.

Use the multiple-choice answer sheet for questions 1-10.

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1 What is the value of $\dfrac{\sqrt{8\cdot 9}+2\cdot 1^2}{\sqrt{8\cdot 9+2\cdot 1^2}}$ correct to 3 significant figures? **1**

(A) $2\cdot 02$
(B) $2\cdot 03$
(C) $2\cdot 026$
(D) $2\cdot 027$

2 What is the exact value of $\sec 30° + \tan 30°$? **1**

(A) $\dfrac{5\sqrt{3}}{6}$

(B) $\dfrac{3\sqrt{3}}{2}$

(C) $\dfrac{5\sqrt{3}}{3}$

(D) $\sqrt{3}$

3 The quadratic equation $2x^2+4x+1=0$ has roots α and β. What is the value of $\alpha^2\beta+\alpha\beta^2$? **1**

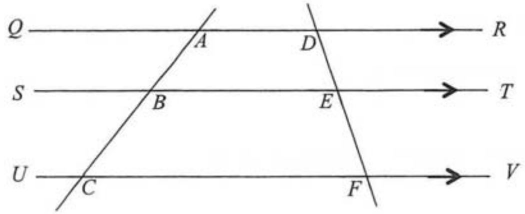
(A) -2
(B) -1
(C) 1
(D) 2

4 The function $f(x)$ is defined by $f(x)=\begin{cases} 4^x, & x\le 1 \\ \dfrac{4}{x}, & x>1 \end{cases}$. What is the value of $f(0\cdot 5)+f(2)$? **1**

(A) 4
(B) 10
(C) 18
(D) 24

Marks

5 In the diagram below $QR \parallel ST \parallel UV$. ABC and DEF are transversals such that $AB = 4$ cm , $BC = 6$ cm and $DF = 8$ cm . What is the length of DE ? 1



NOT TO SCALE

(A) 2·8 cm

(B) 3 cm

(C) 3·2 cm

(D) 3·4 cm

6 Find the equation of the straight line making an angle of 135° with the positive direction of the x-axis and passing through the point (2, –1). 1

(A) $y = x - 3$

(B) $y = -x + 1$

(C) $y = x - 1$

(D) $y = -x + 3$

7 What is the solution of the equation $2^x = 5$? 1

(A) $x = \log_e 5 + \log_e 2$

(B) $x = \log_e 5 - \log_e 2$

(C) $x = \log_e 5 \times \log_e 2$

(D) $x = \dfrac{\log_e 5}{\log_e 2}$

Marks

8 For $k \neq 0$, what is the limiting sum of the geometric series 1

$$k + \frac{k}{1+k^2} + \frac{k}{\left(1+k^2\right)^2} + \frac{k}{\left(1+k^2\right)^3} + \dots \ ?$$

(A) $\dfrac{1}{1+k^2}$

(B) $\dfrac{k^2}{1+k^2}$

(C) $\dfrac{1+k^2}{k}$

(D) $\dfrac{1+k^2}{k^2}$

9 After t hours the number $N(t)$ of individuals in a population is given by $N(t) = 100e^{kt}$ for some constant $k > 0$. After 1 hour there are x individuals in the population. What is the number of individuals in the population after 2 hours? 1

(A) $\dfrac{x}{100}$

(B) $\dfrac{x^2}{100}$

(C) $100x$

(D) $100x^2$

10 A sector of a circle of radius r cm contains an angle of θ radians at the centre of the circle. The sector has area 50 cm^2 . Which of the following is NOT an expression for the perimeter P cm of the sector? 1

(A) $P = r(2+\theta)$

(B) $P = 2r + \dfrac{100}{r}$

(C) $P = \dfrac{20}{\sqrt{\theta}} + 10\sqrt{\theta}$

(D) $P = \dfrac{50(2+\theta)}{r\theta}$

Section II

90 Marks
Attempt Questions 11-16
Allow about 2 hours and 45 minutes for this section.

Answer the questions in writing booklets provided. Use a separate writing booklet for each question. In Questions 11-16 your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) **Use a separate writing booklet.** **Marks**

(a) Express in simplest form with rational denominator $\frac{\sqrt{5}}{3+\sqrt{5}}$. **2**

(b) Solve the inequality $|2x-1|>3$. **2**

(c) Find $\lim_{x\to 2}\frac{x^2-4}{2x-4}$. **2**

(d)(i) If $y=\tan 2x$ find $\frac{dy}{dx}$. **1**

(ii) If $y=e^{x^2}+4\sqrt{1-x}$ find $\frac{dy}{dx}$. **2**

(e) Find in simplest exact form the equation of the tangent to the curve $y=x^2\log_e x$ at the point (e, e^2) on the curve. **3**

(f) The region bounded by the curve $y=\frac{1}{2x+1}$ and the x axis between $x=0$ and $x=2$ is rotated through one revolution about the x axis. Find in simplest exact form the volume of the solid formed. **3**

Question 12 (15 marks) **Use a separate writing booklet.** **Marks**

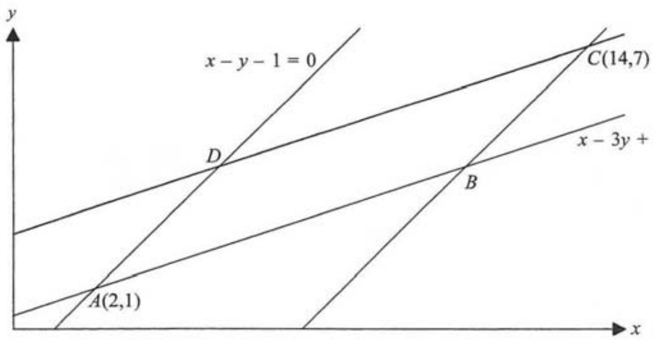
(a) Find the focus and the directrix of the parabola $(x-3)^2=8(y-1)$. **2**

(b) Find in simplest form $\frac{d}{dx}\left(\frac{\cos x}{1-\sin x}\right)$. **2**

(c) Sketch the graph of the function $f(x)=\sqrt{x}-2$ showing the intercepts on the axes. **2**

(d) A curve has gradient function $\frac{dy}{dx}=\frac{x^2}{3}+\frac{3}{x^2}$ and passes through the point $(3,3)$. Find the equation of the curve. **3**

(e)



In the diagram, $A(2,1)$ and $C(14,7)$ are two vertices of a parallelogram $ABCD$. The side AB has equation $x-3y+1=0$ and the side AD has equation $x-y-1=0$.

(i) Find the equation of the side BC. **1**

(ii) Find the coordinates of the point B. **2**

(iii) Find in simplest exact form the area of the parallelogram $ABCD$. **3**

Question 13 (15 marks) **Use a separate writing booklet.** **Marks**

(a) The quadratic equation $2x^2 - 3x + 8 = 0$ has roots α and β.
Find the value of: 1

(i) $\alpha + \beta$ 1

(ii) $\alpha\beta$

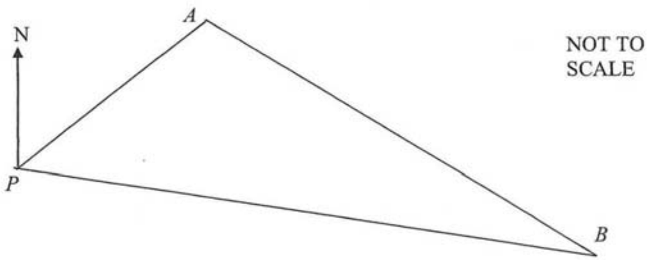
(b) Find $\int \tan^2 x \, dx$. 2

(c) Find any values of k such that $1, \log_e k, 4$ are the first three terms of a geometric progression. 3

(d)(i) Show that $\frac{d}{dx}\left(\log_e \tan x\right) = \frac{1}{\cos x \sin x}$. 2

(ii) Hence find in simplest exact form the value of $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{2}{\cos x \sin x} dx$. 2

(e)



In the diagram a yacht sails 640 metres from point P to point A on a bearing of 050°. It then sails 960 metres from point A to point B on a bearing of 120°.

(i) Find the distance of point B from point P correct to the nearest metre. 2

(ii) Find the bearing of point B from point P correct to the nearest degree. 2

Question 14 (15 marks) **Use a separate writing booklet.** **Marks**

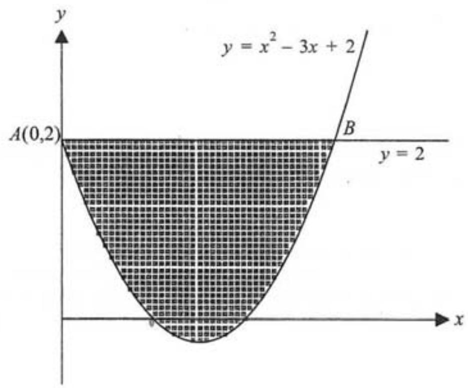
(a) Find the set of values of x for which the curve $y = 3x^2 - x^3$ is concave up. 2

(b) Find in simplest exact form the value of $\int_0^{\log_e 3} \frac{e^x}{e^x + 1} dx$. 3

(c) Find the coordinates and the nature of the stationary point on the curve $y = x + \frac{4}{x^2}$. 3

(d) Find the volume of the solid formed when the semi-circle $y = \sqrt{r^2 - x^2}$ is rotated about the y-axis. 3
(Where r is the radius of the semi-circle.)

(e)



In the diagram the parabola $y = x^2 - 3x + 2$ and the line $y = 2$ intersect at the points $A(0, 2)$ and B.

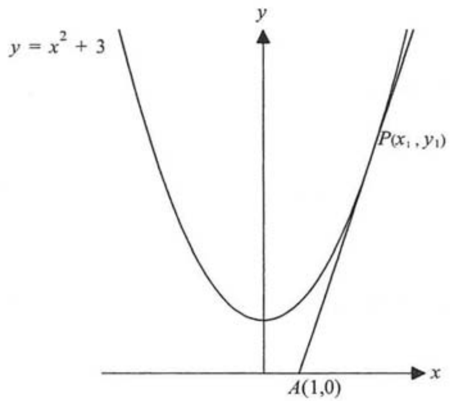
(i) Find the x coordinate of the point B. 1

(ii) Find in simplest form the area of the shaded region between the parabola $y = x^2 - 3x + 2$ and the line $y = 2$. 3

Marks

Question 15 (15 marks) **Use a separate writing booklet.**

(a)



In the diagram $P(x_1, y_1)$, where $x_1 > 1$, is a point on the parabola $y = x^2 + 3$. **3**

The tangent to the parabola at the point P passes through the point $A(1,0)$.

By finding the gradient of AP in two different ways, find the value of x_1.

(b)(i) Solve the equation $1 + 2\sin x = 0$ for $0 \le x \le 2\pi$. **2**

(ii) Sketch the curve $y = 1 + 2\sin x$ for $0 \le x \le 2\pi$ showing clearly the coordinates of the endpoints and the maximum and minimum points. **2**

(c) *Oztown* had a 25 year house building program starting at the beginning of 1991 and finishing at the end of 2015. The number of houses built each calendar year follows an arithmetic progression with first term a and common difference d. 1900 houses were built in the year 2000 and 1100 houses were built in the year 2010.

(i) Find the values of a and d. **3**

(ii) Find the total number of houses built over the 25 years. **1**

(d) A particle is moving in a horizontal straight line. At time t seconds it has displacement x metres to the right of a fixed point O on the line given by $x = t(t-3)^2$, velocity $v\ \text{ms}^{-1}$ and acceleration $a\ \text{ms}^{-2}$.

(i) Find expressions for v and a in terms of t. **2**

(ii) Find when the particle is moving towards O. **1**

(iii) Find when the particle is moving towards O and slowing down. **1**

Marks

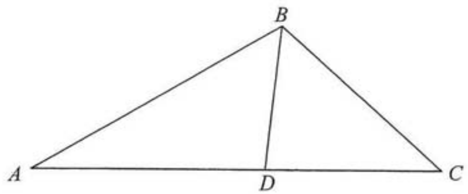
Question 16 (15 marks) **Use a separate writing booklet.**

(a) A water tank containing 10 000 litres of water is being emptied. At time t minutes after it starts to empty the rate R litres/minute at which it is emptying is given by $R = 100e^{-0.01t}$.

(i) Show that the quantity Q litres of water remaining in the tank at time t minutes after it starts to empty is given by $Q = 10000e^{-0.01t}$. **2**

(ii) Find in simplest exact form the time taken for the tank to half empty and the rate at which the tank is emptying then. **2**

(b) In the diagram $\angle DBC = \angle DBA = x°$, $AB = c$, $AC = b$, $BC = 2a$ and $DB = DC = d$.

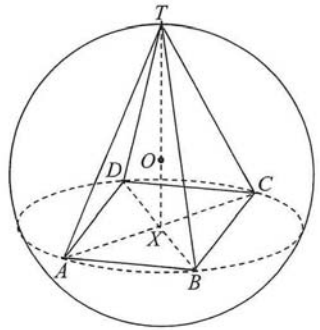


NOT TO SCALE

(i) Show that $\Delta ABD \ ||| \ \Delta ACB$. **2**

(ii) Hence show that $(a+c)^2 = a^2 + b^2$. **3**

(c)



In the diagram the square $ABCD$, whose diagonals AC and BD meet at X, is the base of a right, square-based pyramid with apex T which is inscribed in a sphere of radius 1 metre with centre O, so that the vertices of the pyramid touch the inside of the sphere, and $OX = x$ metres. Given that the volume of a pyramid is $\frac{1}{3}$ *area of base* $\times$ *height*:

(i) Show that the volume V m^3 of the inscribed pyramid is given by $V = \frac{2}{3}(1 + x - x^2 - x^3)$. **3**

(ii) Hence find the maximum volume of the pyramid. **3**

2017 mathematics Trial Answers.

1. $\frac{\sqrt{8.9+2.1^2}}{\sqrt{8.9+2.1}} = \frac{7.3933}{3.6483} = 2.03$ to 3 sig fig (B)

2. $\sec 30° + \tan 30° = \frac{2}{\sqrt3} + \frac{1}{\sqrt3} = \frac{3}{\sqrt3} = \sqrt3$ (D)

3. $\alpha^2\beta + \alpha\beta^2 = \alpha\beta(\alpha+\beta) = \frac12 \times (-2) = -1$ (B)

4. $f(0.5) + f(2) = 4^{\frac12} + \frac42 = 2+2 = 4$ (A)

5. $\frac{AB}{AC} = \frac{DE}{DF}$ $\quad \frac{4}{10} = \frac{DE}{8}$

$DE = 3.2cm$ (C)

6. $m = \tan\theta$ $\quad y - y_1 = m(x - x_1)$

$= \tan 135°$ $\quad y + 1 = -1(x-2)$

$= -\tan 45°$ $\quad y + 1 = -x + 2$

$= -1$ $\quad y = -x + 1$ (B)

7. $2^x = 5$ $\quad \therefore x = \frac{\log_e 5}{\log_e 2}$

$x \log_e 2 = \log_e 5$ (D)

8. $S_\infty = \frac{a}{1-r} = \frac{k}{1 - \frac{1}{1+k^2}}$

$= \frac{k}{\frac{1+k^2-1}{1+k^2}}$

$= \frac{k(1+k^2)}{k^2}$ (C)

9. $N(1) = x$ $\quad x = 100e^{-k}$

$\therefore N(2) = 100e^{-2k}$

$= 100(e^{-k})^2$ (B)

$= \frac{(100e^{-k})^2}{2} = \frac{x^2}{100}$

10. $\frac12 r^2\theta = 50$

$\therefore r\theta = \frac{100}{r}$ and $r^2 = \frac{100}{\theta}$

$\theta = \frac{100}{r^2}$ $\quad r = \sqrt{\frac{100}{\theta}}$

also $P = 2r + r\theta$

$\therefore P = r(2+\theta)$ [A] or $P = 2r + \frac{100}{r}$ [B]

$= \sqrt{\frac{100}{\theta}}(2+\theta)$

$= \frac{20}{\sqrt\theta} + 10\sqrt\theta$ [C]

$\therefore$ (D)

Question 11

a) $\frac{\sqrt5}{3+\sqrt5} = \frac{3\sqrt5 - 5}{9-5} = \frac{3\sqrt5-5}{4}$

b) $2x - 1 < -3$ or $2x - 1 > 3$

$2x < -2$ $\quad 2x > 4$

$x < -1$ $\quad x > 2$

c) $\lim_{x\to2} \frac{x^2-4}{2x-4}$

$= \lim_{x\to2} \frac{(x-2)(x+2)}{2(x-2)}$

$= \lim_{x\to2} \frac{x+2}{2} = \frac{2+2}{2}$

$= 2$

d) i. $y = \tan 2x$

$\frac{dy}{dx} = 2\sec^2 2x$

ii. $y = e^{x^2} + 4\sqrt{1-x}$

$\frac{dy}{dx} = 2xe^{x^2} + 4\{\frac{-1}{2}(1-x)$

$= 2xe^{x^2} - \frac{2}{\sqrt{1-x}}$

e) $y = x^2 \log_e x$

$\frac{dy}{dx} = 2x.\log_e x + x^2 \times \frac1x$

at $x = e$

$m = 2e\log_e e + e^2 \times \frac1e$

$m = 3e$

$y - e^2 = 3e(x - e)$

$y - e^2 = 3ex - 3e^2$

$y = 3ex - 2e^2$

f) $\pi\int_0^2 \frac{1}{(2x+1)^2}dx$

$= \pi\int_0^2 (2x+1)^{-2}$

$= \pi\left[\frac{(2x+1)^{-1}}{-2}\right]_0^2$

$= \pi\left[\frac{-1}{2(2x+1)}\right]_0^2$

$= \pi\left[\frac{-1}{10} + \frac12\right]$

$= \frac{2\pi}{5}u^3$

Question 12

a) Vertex $(3,1)$

$a = 2$

Focus $(3,3)$

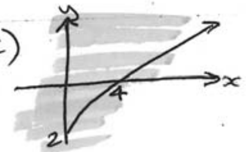
Directrix $y = -1$

b) $\frac{d}{dx}\frac{\cos x}{1-\sin x} = \frac{(1-\sin x).-\sin x - \cos x.-\cos x}{(1-\sin x)^2}$

$= \frac{-\sin x + \sin^2 x + \cos^2 x}{(1-\sin x)^2}$

$= \frac{1-\sin x}{(1-\sin x)^2} = \frac{1}{1-\sin x}$

c)



d) $y = \frac{x^3}{9} + \frac{3}{x} + c$ (at 3,3)

$3 = 3 - 1 + c$

$c = 1$

$y = \frac{x^3}{9} - \frac{3}{x} + 1$

e) i. $m_{BC} = m_{AD} = 1$ $\quad (14,7)$

$y - 7 = 1(x - 14)$

$y = x - 7$

ii) $x - 3y + 1 = 0$

$y = x - 7$

$x - 3(x-7) + 1 = 0$

$-2x = -22$

$x = 11$

$y = 4$ $\quad (11,4)$

iii) $AB^2 = (11-2)^2 + (4+1)^2 = 9^2 + 3^2 = 3^2(3^2+1) = 9$

$AB = 3\sqrt{10}$

$\perp$ distance from C to AB $d = \frac{|14 - 3\times7 + 1|}{\sqrt{1^2 + (-3)^2}}$

$= \frac{6}{\sqrt{10}}$

hence Area $= 3\sqrt{10} \times \frac{6}{\sqrt{10}} = 18u^2$

Question 13

a) for $2x^2 - 3x + 8 = 0$

$a=2 \quad b=-3 \quad c=8$

$\therefore \alpha+\beta = \frac{-b}{a} = \frac{3}{2}$

$\alpha\beta = \frac{c}{a} = \frac{8}{2} = 4$

b) $\int \tan^2 x \, dx = \int (\sec^2 x - 1) dx$

$= \tan x - x + c$

c) for $1, \log_e k, 4$

$\frac{\log_e k}{1} = \frac{4}{\log_e k}$

$(\log_e k)^2 = 4$

$\log_e k = \pm 2$

$\therefore k = e^2 \text{ or } e^{-2}$

d) $\frac{d}{dx} \log_e(\tan x) = \frac{\sec^2 x}{\tan x}$

$= \frac{1}{\tan x} \times \sec^2 x$

$= \frac{\cos x}{\sin x} \times \frac{1}{\cos^2 x}$

$= \frac{1}{\sin x \cos x}$

ii) $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{2}{\sin x \cos x} dx = 2\left[\log_e \tan x\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$

$= 2(\log_e \sqrt{3} - \log_e 1)$

$= 2(\frac{1}{2}\log_e 3 - 0)$

$= \log_e 3$

e) $PB^2 = 640^2 + 960^2 - 2 \times 640 \times 960 \times \cos 110$

$PB^2 = 1751474.35 m$

$PB = 1323 m$

find $\angle APB$ $\quad \frac{\sin P}{960} = \frac{\sin 110}{1323}$

$\sin P = 42.989°$

$\approx 43°$

Bearing $50 + 43 = 093°$

Question 14

a) $y = 3x^2 - x^3$

$\frac{dy}{dx} = 6x - 3x^2$

$\frac{d^2y}{dx^2} = 6 - 6x$

$\frac{d^2y}{dx^2} > 0$ for $x < 1$

$\therefore$ concave up for $x < 1$

b) $\int_0^{\log_e 3} \frac{e^x}{e^x + 1} dx$

$= \left[\log_e(e^x + 1)\right]_0^{\log_e 3}$

$= \log_e(3+1) - \log_e 2$

$= \log_e 4 - \log_e 2$

$= \log_e 2$

c) $y = x + \frac{4}{x^2}$

$\frac{dy}{dx} = 1 - 8x^{-3}$

$\frac{dy}{dx} = 0$ for stat pt

$0 = 1 - 8x^{-3}$

$\frac{8}{x^3} = 1$

$8 = x^3$

$x = 2$

$y = 3$

$\frac{d^2y}{dx^2} = 24x^{-4}$

$\frac{d^2y}{dx^2} = \frac{24}{x^4}$

at $x = 2$ $\frac{d^2y}{dx^2} = \frac{24}{16} > 0$

$\therefore$ minimum.

d) Around y-axis

$y^2 = r^2 - x^2$

$x^2 = r^2 - y^2$

$V = \pi \int x^2 \, dy$

$= \pi \int r^2 - y^2 \, dy$

$= \pi \left[r^2 y - \frac{y^3}{3}\right]_0^r$

$= \pi \left[r^3 - \frac{r^3}{3}\right]$

$= \frac{2\pi r^3}{3}$

e) i) $2 = x^2 - 3x + 2$

$0 = x^2 - 3x$

$0 = x(x-3)$

$x = 0$ or $x = 3$

$\therefore B(3, 2)$

ii) Shaded Area

$= \int_0^3 \{2 - (x^2 - 3x + 2)\} dx$

$= \int_0^3 (3x - x^2) dx$

$= \left[\frac{3}{2}x^2 - \frac{1}{3}x^3\right]_0^3$

$= \frac{3}{2} \times 9 - \frac{1}{3} \times 27$

hence shaded area is $4\frac{1}{2} u^2$

Question 15

a) for $y = x^2 + 3$

$\frac{dy}{dx} = 2x$ $\quad \therefore$ tangent has gradient 2,

gradient of AP is $\frac{y_1}{x_1 - 1} = \frac{x_1^2 + 3}{x_1 - 1}$

$2x_1 = \frac{x_1^2 + 3}{x_1 - 1}$

$2x_1^2 - 2x = x_1^2 + 3$

$x^2 - 2x_1 - 3 = 0$

$(x_1 - 3)(x_1 + 1) = 0$

$x_1 = 3$ or $x_1 = -1$

but $x > 1$

so $x_1 = 3$

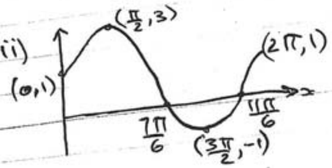
b) i) $2\sin x = -1$

$\sin x = -\frac{1}{2}$

$x = 210°, 330°$

$\frac{7\pi}{6}, \frac{11\pi}{6}$

ii)



c) $1900 = a + 9d$

$1100 = a + 19d$

i) $800 = -10d$

$d = -80$

$1900 = a + 9 \times -80$

$+720$

$a = 2620$

~~$x = t$~~

ii) $S_{25} = \frac{n}{2}(2a + (n-1)d)$

$= \frac{25}{2}(5240 + 24 \times -80)$

$= \frac{25}{2} \times 3320$

$= 41,500$ houses built

d) $x = t(t-3)^2$

i) $\dot{x} = 1(t-3)^2 + t.2(t-3)$

$= (t-3)^2 + 2t^2 - 6t$

$= t^2 - 6t + 9 + 2t^2 - 6t$

$= 3t^2 - 12t + 9$

$\ddot{x} = 6t - 12$

ii) ~~$t=2$~~

iii) $t=0$ $x=0$ $V>0$

particle initially moving right from O

$t > 0 \Rightarrow x > 0$

∴ particle moves towards O for V… ie for $1 < t < 3$

iii Particle moves left and slows down for $v<0$ and $a>0$

∴ $2 < t < 3$

Question 16

a) $\frac{dQ}{dt} = -100e^{-0.01t}$

i) $Q = \frac{-100}{-0.01}e^{-0.01t} + c$

$= 10000e^{-0.01t} + c$

$t=0$, $Q=10000$ } $c=0$, $Q = 10000e^{-0.01t}$

ii) $Q = 5000 \Rightarrow e^{-0.…}$

$e^{0.01t} = 2$

$0.01t = \ln 2$

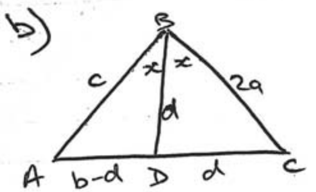
∴ $t = 100\ln 2$ an…

$R = 100e^{-0.…}$

$= 100 \times \frac{1}{2}$

Tank is half full/e… after $100\ln 2$ minute… and is then emptying … a rate of 50L/m…

b)



i) In ΔBDC $\angle DCB = x$ (base ∠'s in isosceles Δ are equal)

then in ΔABD, ΔACB

$\angle ABD = \angle ACB = x$ ~~(∠ACB…)~~

$\angle BAD = \angle CAB$ (common angle)

∴ $\Delta ABD \,|||\, \Delta ACB$ (equiangular)

ii) $\frac{AD}{AB} = \frac{AB}{AC} = \frac{BD}{CB}$ (sides in proportion in similar Δ's)

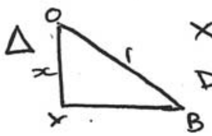
∴ $\frac{b-d}{c} = \frac{c}{b} = \frac{d}{2a}$

∴ $c^2 = b^2 - bd$

and $2ac = bd$

∴ $a^2 + 2ac + c^2 = a^2 + b^2$

$(a+c)^2 = a^2 + b^2$

c) in Δ 

$XB = \sqrt{1-x^2}$

$DB = 2\sqrt{1-x^2}$

$AC = 2\sqrt{1-x^2}$

i) Area ABCD (rhombus) $= \frac{1}{2} AC \times DB$

$= \frac{1}{2} \times 2\sqrt{1-x^2} \times 2\sqrt{1-x^2}$

$= 2(1-x^2)$

height of XT $= OX + OT$

$= x + 1$

$V = \frac{1}{3} \times 2(1-x^2) \times (x+1)$

$= \frac{2}{3}(x + 1 - x^3 - x^2)$

ii) $\frac{dV}{dx} = \frac{2}{3}(1 - 2x - 3x^2)$

$= -\frac{2}{3}(3x-1)(x+1)$

$\frac{d^2V}{dx^2} = \frac{2}{3}(-2-6x)$

$= -\frac{4}{3}(1+3x)$

since $0 < x < 1$ $\frac{dV}{dx} = 0$ and $x = \frac{1}{3}$

$\frac{d^2V}{dx^2} < 0$

hence $V_{max} = \frac{2}{3}\left(1 + \frac{1}{3} - \frac{1}{9} - \frac{1}{27}\right) = \frac{64}{81}$

Maximum Volume is $\frac{64}{81}$ m^3